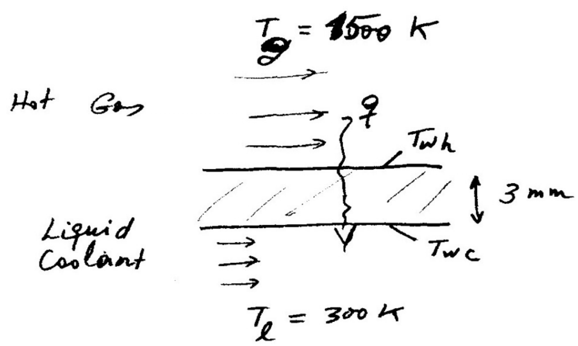
Problem 1

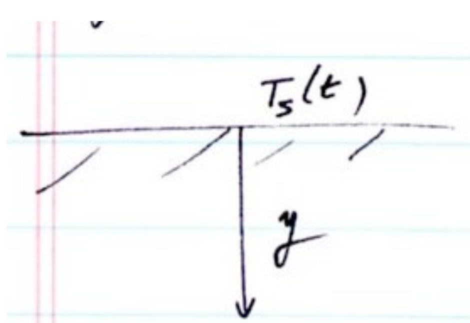


Consider a situation as sketched. The wall is made of steel, with a thermal conductivity K = 40 W/m/K. The gas is air at a pressure of 50 atm, and flows at a velocity of 700 m/s. The liquid is water, flowing at 60 m/s. The friction coefficients are estimated as $c_f = 0.0011$ on the gas side, and $c_f = 0.0033$ on the liquid side.

Calculate the steady state heat flux q and the temperatures T_{wh}, T_{wc}.

Problem 2

The earth's surface is exposed to yearly variations of air temperature, of the form



$$T_s = \overline{T} + \Delta T \sin \omega t \qquad (1)$$

where $\overline{T}$ is the mean temperature, ΔT is the peak seasonal variation, say 20K in temperate latitudes, and

$$\omega = \frac{2\pi}{1\ yr} = \frac{2\pi}{3.15 \times 10^7} s^{-1}.$$

The soil has a thermal conductivity $k \cong 1.6\ w/m/K$, a density $\rho \cong 2000 Kg/m^3$ and a specific heat $c \cong 2000 \frac{J}{KgK}$.

(a) Assume the temperature distribution is of the form $T - \overline{T} = \text{Re}\left[A\, e^{i(\omega t - ky)}\right]$ where A is a complex amplitude factor, and Re [ ] means the real part of a complex number. Substitute into the transient heat conduction equation and calculate k as $k = \pm\sqrt{-i\frac{\omega}{\alpha}} = \pm(1-i)\sqrt{\frac{\omega}{2\alpha}}$. Show that the lower sign leads to exponential divergence with y, so select the upper sign. Choose the constant A so as to reproduce the variation $T_s(t)$ on the surface. Your solution should be

$$T(y,t) = \overline{T} + \Delta T\, e^{-\sqrt{\frac{\omega}{2\alpha}}y} \sin\left(\omega t - \sqrt{\frac{\omega}{2\alpha}}y\right) \qquad (2)$$

(b) At what depth is the amplitude of the temperature oscillation reduced to $\pm 1K$? If peak surface temperature happens on August 1, when does it happen at that depth?